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Simulation of object motion in a bubbling fluidized bed using a Monte Carlo method

L.M. Garcia-Gutierrez *, A. Soria-Verdugo, N. Garcia-Hernando, U. Ruiz-Rivas

Energy Systems Engineering Group, Department of Thermal and Fluids Engineering, Carlos III University of Madrid, Avda. Universidad 20, 28911 Leganés Madrid, Spain

Abstract: The motion of a large neutrally buoyant object immersed in a 2D bubbling fluidized bed was simulated using a Monte Carlo method. The object vertical trajectory within the bed was simulated for a range of dimensionless gas velocities using a simple 1D model. The main characteristics of the object motion were obtained from the trajectory simulation and compared with experimental evidence giving good results. On a second step, the time scale of the motion is introduced in the simulated data by means of well known 2D correlations for the bubble and dense phase velocity. The circulation time of an object (from the instant when it leaves the freeboard and sinks in the dense phase till the moment it reappears back in the surface) was then obtained and compared with experimental data, showing a general agreement. Finally, an extrapolation for a 3D fluidized bed was made following a similar procedure.

Keywords: Monte Carlo, Object motion, Gas fluidization, Bubbling bed, Circulation time, Sinking and rising velocity.

1. Introduction

Most of Bubbling Fluidized Beds (BFB) applications involve the motion of large objects within the bed. The objects might be fuel particles, catalysts, reactants, agglomerates, etc. These objects will have a circulation in the bed mainly determined by sinking forces related with the dense phase motion, rising forces related with the bubbles and buoyancy forces. The buoyancy effects may or may not be significant depending on the bed and object characteristics. The object capability to move throughout the whole bed or stay in a restricted zone will have a paramount effect on key parameters for the performance of the reactor as the reaction time, the existence of hot or cold spots, or the behavior of agglomerates.

* Corresponding author. Tel.: +3491624 8884; fax: +3491624 9430.
E-mail address: lmgarcia@ing.uc3m.es (L.M. Garcia-Gutierrez).

Several works have focused on the motion of an object within a BFB, either as a tracer of the bed material or as a large and lighter or denser object with a relative motion to the dense phase. Most of them are based on experimental analysis, including experiments in 2D and 3D beds, using different measurement techniques. The first studies concerning a large object motion were reviewed by Kunii and Levenspiel (1991). Rios et al. (1986) studied the motion of large objects in 2D and 3D beds and discussed the sinking and rising processes. Pallarès and Johnsson (2006) analyzed the motion of a phosphorescent particle in a 2D bed for several width height configurations. Nienow et al. (1978), Lim and Agarwal (1994), and Rees et al. (2005) studied the sinking and rising process and their characteristic velocities, showing a good agreement for the sinking velocity but a poorer one for the rising velocity. Several of the mentioned works also dealt with the incidence of buoyant forces.

The motion of a large object is quite different than that of a dense phase particle, the main differences being associated with the rising motion. A dense phase particle rises in the bubble wake directly to the bed surface. But when the object is far larger than the bed material (like the case of a fuel particle), a detachment of the object from the bubble is often observed. Rios et al. (1986) found that a large object was not lifted to the bed surface directly by the action of a single bubble, but rather raised to the bed by the action of several bubbles, following a rising path composed of multiple jumps that they called jerks. They reported that the mean rising velocity of objects was lower than the velocity of bubbles due to this fact. The experimental works of Nienow et al. (1978), Lim and Agarwal (1994), and Rees et al. (2005) present similar results. These works define a rough estimate for the mean rising velocity of objects that varies, for different authors, between 10% and 30% of the mean bubble velocity along the bed.

In a previous work directly linked to this study, Soria Verdugo et al. (2011a) also found the multiple jump behavior in the rising path of objects. The mean sinking and mean rising velocity were calculated experimentally for the same object used in the simulations now presented. The results were compared with the dense phase velocity using the Kunii and Levenspiel correlation, and with the bubble velocity using the Shen correlation. The results obtained showed that the mean sinking velocity of the object was quite the same as the dense phase velocity and the rising velocity of the object represented the 20% of the bubble velocity, which is in accordance with the results found previously by other authors. These results are used as inlet data to the model proposed here.

Variations of the object size for large objects (far larger than the bed material) seems to have some effect on the velocities as it modifies the buoyancy forces over the object (Nienow et al., 1978; Rios et al., 1986; Lim and Agarwal 1994; Rees et al., 2005; Soria Verdugo et al., 2011b). Nevertheless, the effect seems to be feeble for a range of object lengths (Soria Verdugo et al., 2011b), and thus the effect can be neglected for objects that have a proper circulation in the bed. In the present work, the results are given for just one object, and no shape, size or density variations are considered.

Soria Verdugo et al. (2011a) also analyzed the distribution of the number of jumps (number of bubbles that raised the object upwards during a cycle) experienced by a neutrally buoyant object in a cycle, going from the bed surface to a certain maximum depth and back to the surface. The probability density function of the number of jumps was found to follow a geometric fitting in the form of Eq. (1)

$$P(N_j) = p(1\ \ p)^{N_j - 1} \quad \text{for} \quad N_j = 1, 2, 3... \tag{1}$$

where N_j is the number of jumps, and p is a parameter given by the fitting. The value of $p = 0.45$ gave a good match with the experimental data.

When an object has started a raising path with a bubble, there should be a probability for it to reach the bed surface in that path and a probability (complementary) to detach from that bubble before arriving to the surface and begin a new sinking path. The form of the probability density function in Eq. (1) allows us to consider the value of such a probability as that of parameter p, being $(1\ \ p)$ the detaching probability. The parameter p obtained from this geometrical fitting is a mean parameter for the whole height of the bed, as the experimental data was obtained for every cycle, disregarding the depth at which each jump began.

The validity of the value of $p = 0.45$ for other objects and bed conditions was analyzed in a second work (Soria Verdugo et al., 2011b), varying the dimensionless gas velocity, the bed height, the dense phase diameter and the object characteristics (length and density) and obtaining similar values in all the cases when a proper circulation was observed. When the buoyancy forces are predominant and the circulation is poor (objects remaining in the bed surface or sinking to the distributor and remaining there) the parameter has no longer any meaning. Therefore, the value of p will be considered constant and equal to 0.45 throughout this work. Note that varying the dimensionless gas velocity and the bed height suppose a variation of the bubble diameter, but no effect was found on parameter p.

The experimental evidence presented in Soria Verdugo et al. (2011a, 2011b) and concerning the mean sinking and rising velocities and the value of the parameter p provide the basis for the Monte Carlo simulation presented herein.

Monte Carlo simulations are often used to characterize statistical parameters in a Fluidized Bed, such as dense phase particle collisions (Huilin et al., 2006), bubble probe interactions (Rüdisüli et al., 2012) or mixing and circulation of solids (Larachi et al., 2003), among others. In this work, it will be used to define and quantify probabilities (such as p in Eq. 1), that are involved in the motion of neutrally buoyant objects within the bed.

2. Experimental setup

Some experimental or theoretical results are needed to provide inlet data for the simulations and for comparison with the simulation results. This is largely achieved by referring to the experiments of Soria Verdugo et al. (2011a) and to well known correlations in the literature. Nevertheless, some experiments have been carried out to cope with specific aspects.

The experimental facilities consisted of two bubbling fluidized beds (BFB): a 2D bed and a lab scale 3D bed. The 2D fluidized bed had 2 m in height, a width of 0.5 m and a thickness of 0.01 m. The height of the packed bed was 0.5 m. The bed material was spherical ballotini particles, with a diameter range of 600 800 μm. The minimum fluidization velocity was measured to be 0.32 m/s and the dimensionless gas velocity (U/U_{mf}) during the experiments was set to 2, 2.5 and 3. The object used during the tests was a cylinder with 0.0064 m of diameter, a length of 0.019 m and a density of 1508 kg/m^3, whereas the bed density was 1560 kg/m^3. Thus, the buoyancy effects can be considered negligible, and the object is further referred to as neutrally buoyant.

Table 1
General information about the experimental setup and other parameters.

2D bed facility	
Bed height [m]	2
Bed width [m]	0.5
Bed thickness [m]	0.01
Fixed bed height [m]	0.5
Minimum fluidization velocity (U_{mf}) [m/s]	0.32
Dimensionless gas velocity (U/U_{mf}) [–]	2; 2.5; 3
3D bed facility	
Bed height [m]	1
Bed diameter [m]	0.192
Fixed bed height [m]	0.192
Minimum fluidization velocity (U_{mf}) [m/s]	0.25
Dimensionless gas velocity (U/U_{mf}) [–]	2; 2.5
Bed material (in both beds): ballotini particles	
Particle diameter [μm]	600–800
Skeletal density [kg/m^3]	2500
Bulk density [kg/m^3]	1560
Object	
Length [m]	0.019
Diameter [m]	0.0064
Density [kg/m^3]	1508
Other paramaters	
Area of the distributor per number of orifices (A_o) [m^2]	$5 \times 10^{\ 5}$
Bubble wake fraction (f_w) [–]	0.18
Constant determined experimentally (λ) [–]	9.86
Constant determined experimentally (Φ) [–]	0.9

Table 1 provides general information about the experimental setup. Further details of the experimental setup and the measurement techniques can be found in Soria Verdugo et al. (2011a).

The lab scale 3D bed had a cylindrical vessel with an inner diameter of 0.192 m and a height of 1 m. The object, the bed material and the bed aspect ratio were the same as the 2D bed. The minimum fluidization velocity was 0.25 m/s and the dimensionless gas velocity (U/U_{mf}) during the experiments was set to 2 and 2.5. The surface of the bed was recorded during 20 min at 30 fps, so the object could be only detected while on the freeboard. A more detailed description of the experimental setup can be found in Soria Verdugo et al. (2011c).

The difference of U_{mf} obtained between a 3D and a 2D bed of similar characteristics has been observed by several authors. Sánchez Delgado et al. (2011) established that U_{mf} increases when decreasing the bed thickness below a certain critical thickness. Above this critical thickness the wall effect is negligible and the behavior is like a 3D bed. According to the value of their critical thickness, in our test some wall effect should exist, affecting the U_{mf}. On the other hand, the values obtained for U_{mf} in both cases (2D and 3D bed) are coherent with their values and those of other authors reviewed there.

3. Monte Carlo simulation

3.1. Numerical method

The Monte Carlo method was used for modeling the behavior of the object in the rising and sinking process and determining the circulation time. The model was based on two external and experimental parameters: the probability of an object in a rising path to arrive at the surface of the bed in a one jump rising (p) and the average probability of the objet in a sinking path to continue sinking (q). The model was 1D and the bed was divided in 10 slices of 0.05 m, between a position defined by the packed bed height (depth=0) and the bottom of the bed (depth=0.5 m).

The simulation was run for 10 million objects. The procedure is schematized in Fig. 1. The first object ($o=1$) starts to sink and arrives to the first slice ($j=1$), where two possibilities appear: to continue sinking or to start a rising path. Thus, a random number between 0 and 1 is generated (N) and the object continue sinking to the next slice ($j=2$, etc.) if the number is below or equal to the external data of the probability to sink in this slice $q(j)$. In that case, another random number (N) is generated and the process is repeated. But if the random number is higher than $q(j)$, the object starts a rising path. In order to state whether the object reaches the bed surface during that rising path or not, a new random number (M) between 0 and 1 is generated and its value compared with the average probability of an object to get to the surface (p) in that rising path. A value of M below or equal to p means that the object will reach the surface, so the cycle is completed and another object ($o=2$) is simulated. Nevertheless, if M is higher than p, the object is considered to detach from the bubble before reaching the bed surface (in a position i between slice j and 1), and begin a new sinking process.

3.2. Inlet data

The simulation is based in some inlet data, which are either experimental data taken from the literature, new experiments presented here or data obtained from well established models. Those are

- the probability to continue sinking when arriving to a certain depth $q(j)$;
- the probability of an object to get to the surface when starting a rising path (p); and
- a parameter that establish at which position (i) the object is detached in its rising path in the case that it is not able to reach directly the surface;
- the sinking and rising velocities.

The first two values can be obtained experimentally the third was modeled and the last two obtained from well known correlations.

The probability of an object to reach directly the surface (p) was experimentally obtained for a 2D bed by Soria Verdugo et al. (2011a, 2011b) as previously stated in the Introduction section. The value of p was found to be 0.45 for a wide range of gas velocities and object densities and sizes.

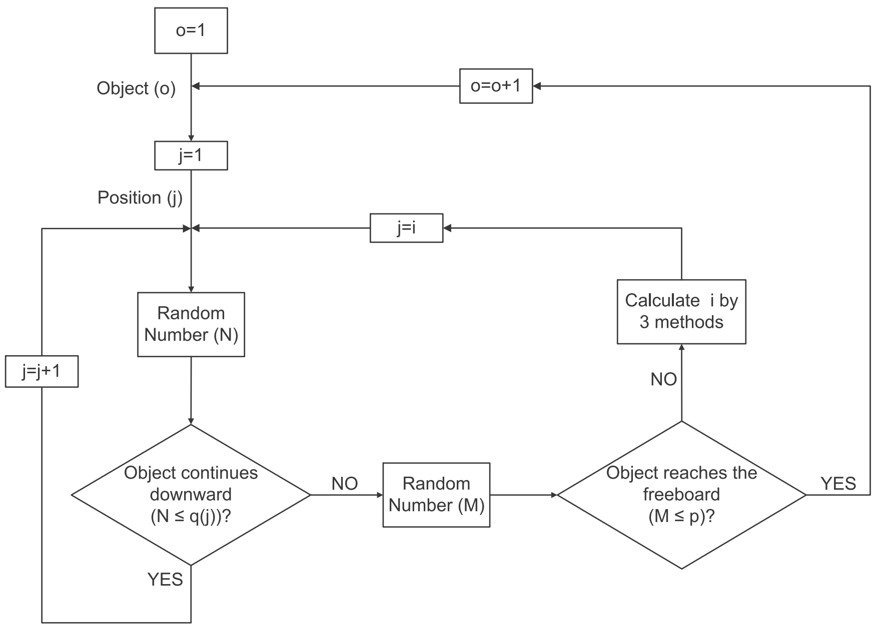

Fig. 1. Scheme of the Monte Carlo model.

Nevertheless, p is, as previously stated in the Introduction section, a mean parameter, and the model demands a local value, $p(j)$, for a jump beginning at a given depth. Therefore, an assump tion is made in the sense that the probability defined by p will be considered the same everywhere in the bed. The experimental evidence shown in Soria Verdugo et al., (2011b), showing a negligible effect on p of varying the bubble diameter, seems to back this assumption. The local parameter should remain near to the mean value, as the main cause of variation as a function of depth is the bubble diameter variation. Therefore, a value of $p(j)=$ 0.45 is assumed throughout the bed and for all cases in the Monte Carlo simulations.

The probability to continue sinking when arriving to a certain depth $q(j)$, was experimentally obtained for the 2D bed conditions stated in the Experimental Setup section. The probability for an object of starting a rising path in each slice is plotted in Fig. 2. The depth of the bed was defined by the opposite of the height, thus a height of 0.5 m corresponded to a depth of 0 m, and the bottom of the bed corresponded to a depth of 0.5 m. It was observed that the average probability of starting a rising path was constant around 20% for most of the bed and then sharply increases up to 100% in the bottom of the bed, a place where an object can only start a rising path.

Finally, the parameter i that establishes at which position the object is detached in its rising path should be obtained. This parameter is unknown and the present experiments cannot supply sufficient statistics for a proper statement. Therefore, several limit cases have been defined. These cases are not used as facts but as limits. An object that rises but do not get to the surface might detach from the bubble that is raising it (a) with more probability at the beginning of the rising path, (b) with equal probability anywhere in the rising path. The actual process should be some where between this two limits.

Therefore, in a first case that will be called *random loss case*, the probability of the object to detach from the bubble is considered constant for any i position between the position where the object starts to rise (j) and the first position underneath the freeboard ($j=1$). The second assumption will be divided in two cases to account for some differences when the rising path begins in the first upper slice. Therefore, in a second case, called *maximum loss 1*, the object was supposed to detach from the bubble immediately (at position $j\;\;1$), except for the particular case of $j=1$ where the object should be released in the same slice ($j=1$). Here, a small variation can be considered, giving a third case called *maximum loss 2*. In this case the object was supposed to detach from the bubble at position $j\;\;1$ for any initial rising position, even for $j=1$. Thus, in these case an object that starts to rise in $j=1$ will arrive to the surface with 100% probability. This contradicts the statement made for p, but was found to be a useful assumption. Eqs. (2) (4) show the three cases

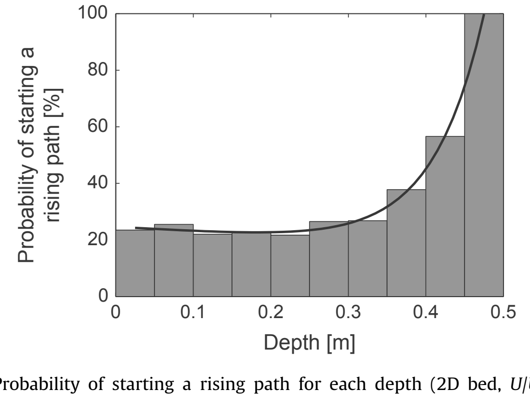

Fig. 2. Probability of starting a rising path for each depth (2D bed, U/U_{mf} 2.5, neutrally buoyant object).

$$\text{Random loss}:\ i = \text{random number}\,[1, j] \tag{2}$$

$$\text{Maximum loss 1}:\ \begin{cases} j \neq 1;\, i = j\;\;1 \\ j = 1;\, i = j \end{cases} \tag{3}$$

$$\text{Maximum loss 2}:\ \forall j;\, i = j\;\;1 \tag{4}$$

where i is at which position the object is detached in its rising path and j denotes the slice number.

From these three parameters, the objects trajectories can be obtained using a Monte Carlo method. But a time scale was not yet available for the proper evaluation of the simulation. This time scale can be introduced by means of a characteristic sinking and rising object velocity for each bed position. The procedure to calculate such velocities was the following. The sinking velocity for a neutrally buoyant object can be considered equal to the dense phase downward velocity, which can be obtained using the Kunii and Levenspiel (1991) correlation (Eq. 5). The diameter of the bubble was calculated using the correlation (Eq. 6) of Shen et al. (2004), and introduced in the correlation of Davidson and Harrison (1963) for the bubble velocity (Eq. 7). The rising velocity for a neutrally buoyant object was generally established to range between a 10% and a 30% of the bubble velocity obtained with Eq. (7) (Nienow et al., 1978; Lim and Agarwal, 1994; Rees et al., 2005). For these work it was assumed a 20% value, as obtained by Soria Verdugo et al. (2011a)

$$v_{dp} = \frac{f_w \delta U_B}{1\;\;\delta\;\;f_w \delta} \tag{5}$$

$$D_B = \left[\;\;\frac{8(2^{3/4}\;\;1)}{\lambda}\right)(U\;\;U_{mf})\;\;h + \frac{\lambda}{\pi(2^{3/4}\;\;1)}\frac{A_0}{T}\right)\right]^{2/3} g^{-1/3} \tag{6}$$

$$U_B = U\;\;U_{mf} + \phi\sqrt{g D_B} \tag{7}$$

where v_{dp} is the downward velocity of the dense phase and depends on the bubble wake fraction, f_w, the bubble fraction in the bed, δ and the bubble velocity, U_B. The bubble diameter, D_B is a function of a constant determined experimentally, λ, the gas velocity, U, the minimum fluidization velocity, U_{mf}, the height over the distributor, h, the area of the distributor per number of orifices A_O and the bed thickness, T. Finally, ϕ is a constant determined experimentally.

3.3. Validation

The numerical method was run for ten million objects and some results were obtained in order to validate the procedure. First, it should be established that the number of objects was sufficient for proper statistics. Fig. 3 shows the median of the circulation time of an object (defined here as the time between the moment when the object leaves the freeboard and that when it reaches back to it), obtained for different runs of the numerical computation and for a varying number of objects being used in the calculation. The results showed that a convergence can be ascer tained for a number of objects larger than one million.

The probabilities p and q were also obtained from the Monte Carlo simulation data, for validation purposes and in order to estimate the effect of the deviation introduced by the assumption in the *maximum loss 2 case* in the value of p. The results for q fit exactly the data of Fig. 2. Concerning p, its value was obtained following the procedure of Soria Verdugo et al. (2011a) and described in the Introduction section. There, the object trajectory within the bed was described by the number of jumps it needs to reach the surface, each jump representing a rising path

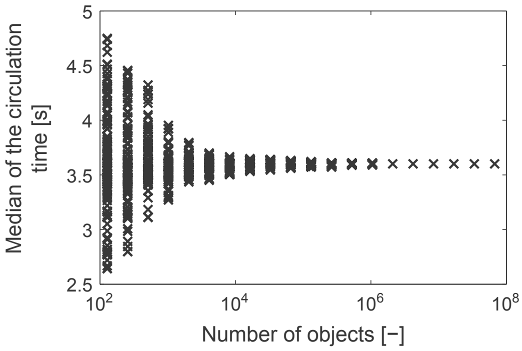

Fig. 3. Median of the circulation time as a function of the number of objects used for the simulation.

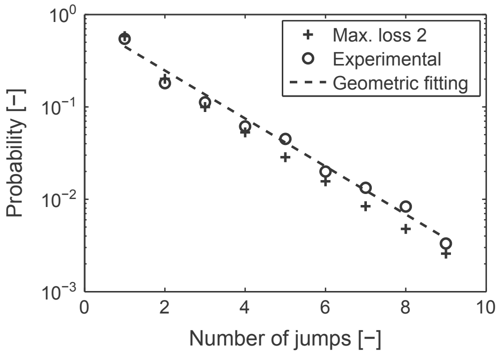

Fig. 4. Probability of a certain number of jumps occurring in a cycle.

associated with a passing bubble that may or may not raise the object directly back to the surface of the bed. Such probability will then follow Eq. (1).

The results for the *random loss case* and the *maximum loss 1 case* exactly represent the $p=0.45$ fitting curve, while the deviation that the *maximum loss 2 case* produced can be observed in Fig. 4. The experimental data for a neutrally buoyant object at $U/U_{mf}=2.5$ presented in Soria Verdugo et al. (2011a), Eq. (1), and the results of the simulation for the *maximum loss 2 case* were plotted together. The results showed that a minor effect exists as the $j=1$ condition increases the probability for $N_j=1$, but it still fits quite well with the experimental results, and notably better for the $N_j=1$ probability.

4. Results and discussion

The simulation provides a complete temporal description of the object trajectories within the bed. In this section, simulation results will be compared with independent experimental results in order to establish the model accuracy in representing the object motion within the bed. First, some parameters of the object motion will be compared with experiments already presented in Soria Verdugo et al. (2011a), and then the circulation time distribution will be presented and compared with experimental results of the same work (in a 2D bed) and with new experimental data obtained in the lab scale 3D bed presented in the Experimental Setup section.

4.1. Object motion within the bed

The probability of finding an object at a certain height and the probability of an object to attain a certain maximum depth throughout its cycle were experimentally obtained as identifying parameters of the object motion in Soria Verdugo et al. (2011a). The former identifies the availability of the fluidized bed to circulate the object throughout the whole bed and the latter characterized the object cycles. In this section, both probabilities were calculated from the Monte Carlo results and were compared with the independent experimental data obtained by Soria Verdugo et al. (2011a).

The procedure for calculating the probability of finding an object at a certain height from the numerical results tried to reproduce the experimental procedure. The acquisition frequency in the experimental tests was set to 1.4 frames per second and the bed was discretized in ten slices, in a similar way than what was previously done for the Monte Carlo simulation. The experiments show the actual position of the object each 0.71 s (1/1.4 fps), but the Monte Carlo results give the position of the object in each passing slice, independent of the time that the object remains in it. Then the probability of an object to be in a certain height can be calculated from the Monte Carlo object trajectory using the local velocity of the object, obtained from the local dense phase velocity or the bubble velocity. In order to compare both probabilities (the experimental and the simulation), the position of the object obtained by the simulation is interpolated for each 0.71 s. These results are shown in Fig. 5(a) for the three simulation cases. A general agreement between experiments and simulation and a similar tendency for the three cases can be observed. At the bottom of the bed and up to a height of 0.3 m, all results show a similar tendency of increasing probability with height, while the simulations give a smaller value than the experimental data. On the other hand, for higher heights than 0.3 m, the experimental data shows an almost constant probability with height while the simulation data shows a continuous increase. This deviation indirectly affects (the results being a probability density function) the values in the bottom of the bed.

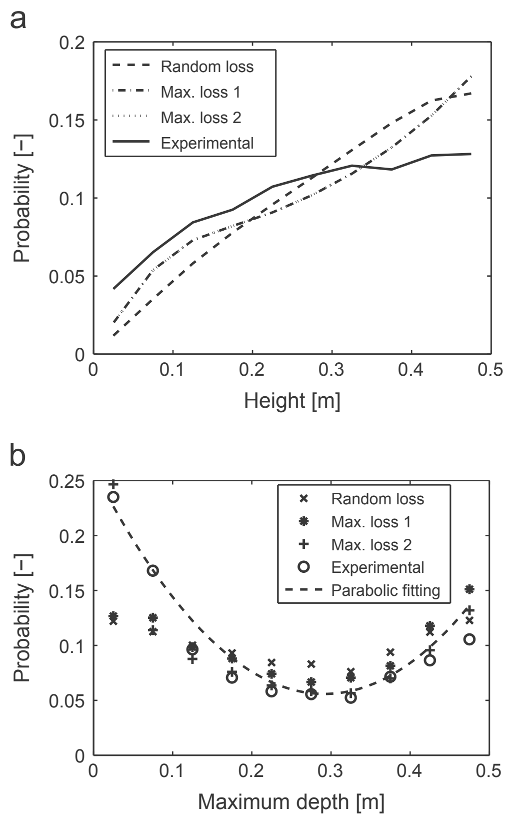

Fig. 5. Probability of (a) finding the object at a certain height and (b) reaching a determined maximum depth in a cycle.

The probability that the object has of attaining a certain maximum depth throughout its cycle is plotted in Fig. 5(b). The previous experimental results showed a parabolic fitting which is presented in the graph. Once again, a similar tendency is observed for the simulated results, but some discrepancies are found near the surface. The results show a minimum of the probability around a depth of 0.3 m in all cases, with values between 0.05 and 0.08, while at the bottom of the bed the probability increases to values between 0.12 and 0.15. For low depths the experimental data show higher probabilities than the simulation, except for the particular case of the result for the *maximum loss 2 case* in the higher slice, where the result is in good accordance with the experimental data. It should be noted that the main difference between *maximum loss 2 case* and the other two cases is that an object starting a rising path in the first slice is forced to attain the surface in the *maximum loss 2 case* and has 55% probability of remaining in the cycle in the other two cases. Therefore, it may be stated that the behavior near the freeboard may differ from that in other regions (and thus from the mean values obtained by the model), weakening the model and causing such differences.

4.2. Circulation time (2D case)

In this section, the circulation time of a neutrally buoyant object immersed in the bed was calculated using the Monte Carlo results and compared with the independent experimental data obtained by Soria Verdugo et al. (2011a). The circulation time could be calculated from the information provided by the simu lated object trajectories with the addition of information of the time scale. This is introduced using well known equations for dense phase and bubble velocities, as established in the Monte Carlo section. For all objects simulated the process starts in the first slice (j=1) and finishes at the freeboard, thus the circulation time is defined as the time that the object spends in a complete a cycle.

The statistical results for the circulation time distribution are presented using box plots. In a box plot, the distribution of the data is represented by several statistical parameters and outlying data: the median, the lower and the upper quartiles (representing 25% and 75% respectively of the population), a confidence interval (the upper limit is the upper quartile plus 1.5 times the inter quartile range and lower limit is the lower quartile minus 1.5 times the inter quartile range), and outliers (corresponding with the data lies out of upper confidence interval limit). The data compar ison was made for three experimental cases defined by the same object and bed characteristics except a varying dimensionless gas velocities (with values of 2, 2.5 and 3). The results are plotted in Fig. 6, comparing the previous experimental results with the data obtained for the *random loss case* simulation. The data of the other cases is not presented in the figure, as they were in good agreement with the *random loss case* results.

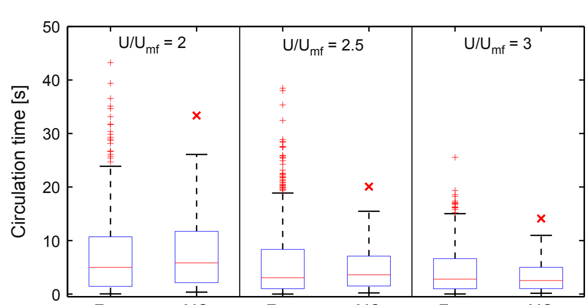

Fig. 6. Box plot of circulation time for three dimensionless gas velocity in the 2D fluidized bed.

The results obtained in the simulation were in agreement with the experimental data either for the general tendency with gas velocity and for each of the three dimensionless gas velocity cases. The simulation was run for 10 million objects and the number of outliers of the circulation time generated for this amount of objects cannot be compared with the number of outliers obtained in the experimental data, where just 1200 cycles are observed. Therefore, the simulation results do not show outliers, but the circulation time that has the same probability as an outlier in the experimental case (1/1200). Throughout this paper, the average value of p is 0.45, as obtained from previous experiments. Never theless, an analysis of the effect of varying p on the simulations (for U/Umf = 2.5 and in the range $0 < p < 1$) was performed, as the value of probability p is a key parameter for the simulation of object motion in fluidized beds. The results showed a dependence of the median of the circulation time distribution on parameter p that is roughly equal to 2/p. Parameters p in the range 0.40 0.55 give good agreement with the presented experimental data pre sented in Fig. 6.

4.3. Circulation time (extrapolation for a 3D case)

In order to evaluate the validity of the simple 1D model established in this work for more complex configurations, a simulation was run considering a 3D experiment. The procedure used for simulating the 3D case is similar to that used in the 2D case. The only difference consists of the equation used for the bubble diameter. Darton equation (Darton et al., 1977) is used in the 3D case, while Shen equation was used in the 2D case. The data obtained was compared with experimental data obtained in the lab scale 3D fluidized bed presented in the Experimental Setup section. The experimental results were obtained recording the freeboard with a zenithal camera and measuring the time lapse between the moment when an object disappears from the surface and the moment when it appears again on the surface. The tests were made for a dimensionless gas velocity of 2 and 2.5. For a dimensionless gas velocity of 3, the object was not clearly detected in the surface due to the amount of dense phase present in the freeboard because of intense bubble eruptions.

The results were compared for dimensionless gas velocity of 2 and 2.5 and plotted in Fig. 7. The simulations are shown only for the *random loss case*, the other two cases giving, again, very similar results. The results show a good agreement, although a wider range of dimensionless gas velocity is necessary. The circulation time shows a slight decrease for increasing dimensionless gas velocity, similarly to the 2D case. The simulation for the 3D case might therefore be considered consistent.

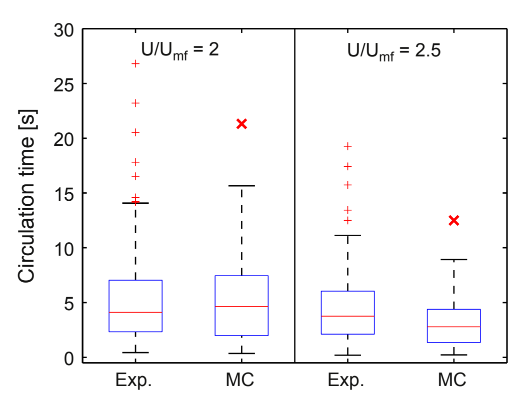

Fig. 7. Box plot of circulation time for two dimensionless gas velocity in the 3D fluidized bed.

5. Conclusions

A one dimensional model has been proposed to describe the motion of a neutrally buoyant object in a 2D bubbling fluidized bed for different dimensionless gas velocities. The model is based on a Monte Carlo method using three representative parameters and the results of the simulation were compared with some experimental data.

The model describes the vertical trajectories of an object within the bed, using statistical parameters to simplify the complex rising and sinking processes. The sinking process is defined by the probability of an object at a certain depth to continue sinking, a parameter that is depth dependent. The rising process is character ized by the probability of a rising object to reach the freeboard in that rising path and, otherwise, its probability of stopping rising and start sinking again at a given depth. These parameters can be either modeled, obtained from the literature or from simple 2D experiments.

The Monte Carlo algorithm was run for 10 million of objects. Convergence can be ascertained above one million objects and the results were tested to check their congruency with the initial data.

The validity of the simulation was tested by comparing its results with experimental results for some independent character istics of the object motion. With such a purpose, the probability distribution for finding an object at a certain height and the maximum depth attained by the object in a cycle were obtained experimentally and calculated from the raw experimental data. The comparison showed in both cases an overall agreement and a similar tendency with depth, while some discrepancies appeared near the freeboard.

The circulation time of an object was calculated using the model results and the velocities of the object in order to introduce a time scale in the vertical trajectories. These velocities were calculated using typical correlations for the dense phase and bubble velocity in a 2D fluidized bed. The results were compared with the experiments showing similar values for all dimensionless gas velocities.

Finally an extrapolation for a 3D fluidized bed was made, using the same simulation results as the 2D case (i.e. extrapolating the validity of the three statistical parameters that determined the sinking and rising behavior) but a 3D correlation for the bubble velocity. The comparison of the circulation time obtained by the method with the measurements showed a good agreement, similar to that obtained for the 2D case although more experi mental data are needed for a proper characterization.

Nomenclature

A_O	Area of the distributor per number of orifices [m^2]
D_B	Bubble diameter [m]
f_w	Bubble wake fraction [ ]
g	Gravity [m/s^2]
h	Height over the distributor [m]
N	Random number [ ]
N_j	Number of jumps needed for the object to rise to the surface of the bed [ ]
M	Random number [ ]
p	Probability of an object to get to the surface [ ]
$P(N_j)$	Probability density function [ ]
$q(j)$	Probability of an object to continue sinking [dimensionless]
T	Bed thickness [m]
U	Superficial gas velocity [m/s]
U_B	Bubble velocity [m/s]
U_{mf}	Minimum fluidization velocity [m/s]
v_{dp}	Downward velocity of the dense phase [m/s]
W	Bed width [m]
δ	Bubble fraction in the bed [ ]
λ	Constant determined experimentally [ ]
ϕ	Constant determined experimentally [ ]

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